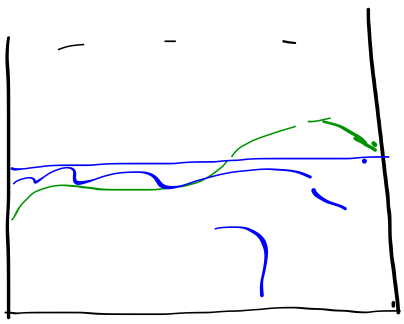
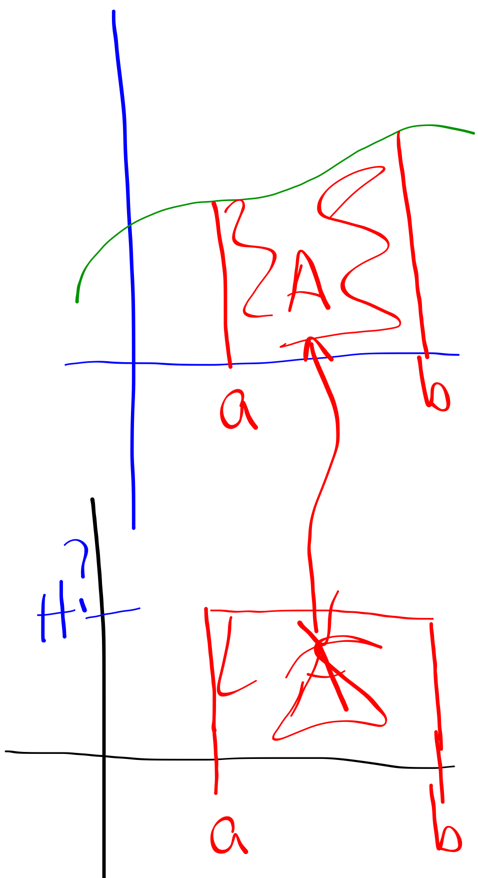
day 71

5.5 The Sand Art Theorem





$$(b-a)H = \int_a^b f(x)\,dx$$

$$H = \frac{1}{b-a}\int_a^b f(x)\,dx$$

H is the "AVERAGE VALUE" of f(x) between a and b

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continuous Average Value ⇒ Mean Value Theorem for Integrals

If a function is continuous on $[a,b]$ then there is a c in (a,b) with $f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$

5.5/1-10

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4.7/47) $\lim_{x \to 0} \csc 6x \sin 7x$

$= \lim_{x \to 0} \frac{1}{\sin 6x} \sin 7x = \lim_{x \to 0} \frac{\sin 7x}{\sin 6x}$

$\underset{=}{LH} \lim_{x \to 0} \frac{7\cos(7x)}{6\cos(6x)} = \frac{7}{6}$

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4.3/25)

$f(x) = x - 3x^{\frac{1}{3}}$ → $f(0) = 0$, $f(-1) = 2$, $f(1) = -2$

$f'(x) = 1 - x^{-\frac{2}{3}} = 1 - \frac{1}{x^{2/3}} = \frac{x^{2/3} - 1}{x^{2/3}}$

Critical Pts

Zero f'

$x^{2/3} - 1 = 0$

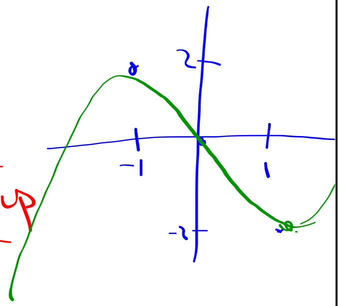
$(x^{1/3} - 1)(x^{1/3} + 1) = 0$

$x = \pm 1$

Und f'

$x^{2/3} = 0 \Rightarrow x = 0$

$f''(x) = \frac{2}{3}x^{-5/3} = \frac{2}{3x^{5/3}}$



sign chart of $f''(x)$: − − − concave down | 0 | + + + concave up

Sign chart of $f'(x)$: + + + (−10) | −1 | − − − ($-\frac{1}{2}$) | 0 | − − − ($\frac{1}{2}$) | 1 | + + + (10)

$1 - x^{-2/3} = f'(x)$

$1 - \frac{1}{\sqrt[3]{100}}$ $\quad 1 - \frac{1}{\sqrt[3]{\frac{1}{4}}}$

End Behavior

$\lim_{x \to \infty} x - 3\sqrt[3]{x} = \infty$

Zeros

$x - 3\sqrt[3]{x} = 0$

$\sqrt[3]{x}\left((\sqrt[3]{x})^2 - 3\right) = 0$

$x = 0$

$(\sqrt[3]{x})^2 = 3$

$\sqrt[3]{x} = \pm\sqrt{3}$

$x = \pm 3\sqrt{3}$ x int

day 71

4.7/51) $\lim_{x\to 0^+} \cot x - \frac{1}{x} = \lim_{x\to 0^+} \frac{\cos x}{\sin x} - \frac{1}{x}$

$$= \lim_{x\to 0^+} \frac{x\cos x - \sin x}{x \sin x}$$

$$\overset{LH}{=} \lim_{x\to 0^+} \frac{\cos x + x(-\sin x) - \cos x}{\sin x + x\cos x}$$

$$= \lim_{x\to 0^+} \frac{-x\sin x}{\sin x + x\cos x}$$

$$\overset{LH}{=} \lim_{x\to 0^+} \frac{-(\sin x + x\cos x)}{\cos x + \cos x + x(-\sin x)} = \frac{0}{2} = 0$$

numerator $\to 0$; denominator $\to 1+1-0$

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